



Calculation Policy

Strong confidence with both mental and written calculations involving the four operations of addition, subtraction, multiplication and division, allows children to fulfil each of the aims of the national curriculum for mathematics:

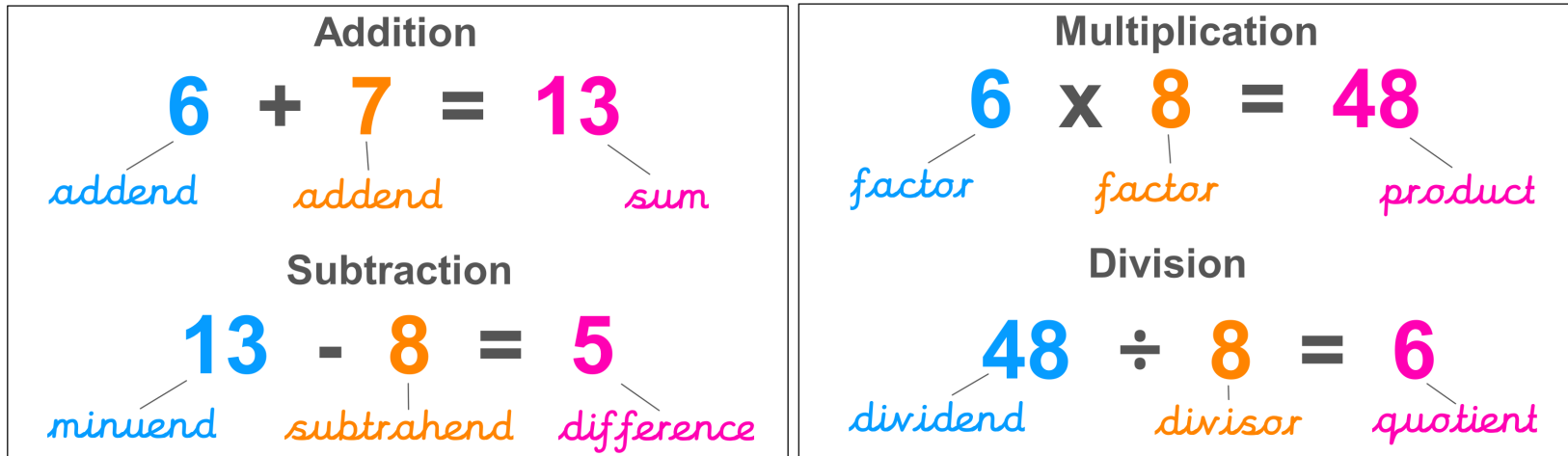
- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

At Hook Junior School, we teach the four operations as inter-connected operations, where addition & subtraction and multiplication & division are the inverse of one another and multiplication is a repeated form of addition and division a repeated form of subtraction. Exploring these relationships between them allows children to develop a deeper understanding of the fundamentals of maths and equip them with the skills required to solve problems in a wide variety of contexts.

The teaching of these, as with other aspects of the maths curriculum, is frequently introduced through a problem and initially explored using concrete resources (such as dienes and place value counters) and pictorial representations before moving to the abstract representation. An example of this progression is shown below beside each written and mental method below.

Vocabulary of the four operations

To help the children to describe patterns within calculations, we use the precise terminology for each part of a calculation. These are shown below and allows statements such as *'in subtraction, if the minuend and subtrahend are both increased or decreased by the same amount, the difference will remain the same'* to be explored.

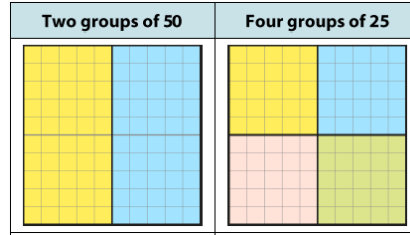


Year 3 – addition and subtraction

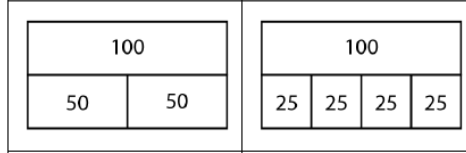
Application of Number Facts

Concrete and Pictorial Representations

Number facts to 100
(These are explored both as additive and multiplicative equations)



25 stickers 25 stickers 25 stickers 25 stickers

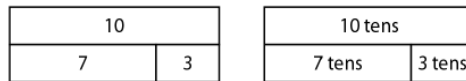
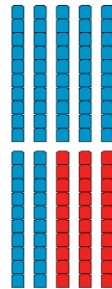


Abstract

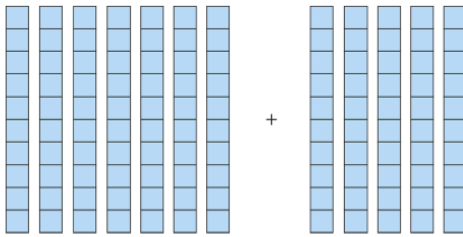
$100 = 50 + 50$	$100 = 25 + 25 + 25 + 25$
$100 = 2 \times 50$	$100 = 4 \times 25$
$100 \div 2 = 50$	$100 \div 4 = 25$
$100 \div 50 = 2$	$100 \div 25 = 4$

$100 = 25 + \square + 25 + 25$
 $100 = 50 + \square$
 $100 - 25 = \square$ $100 - 50 = \square$
 $100 = 4 \times \square$ $2 \times \square = 100$
 $100 \div 4 = \square$ $\square = 100 \div 2$
 $100 - 20 - 20 = \square$
 $\square = 100 - 10 - 10 - 10$

Known addition number facts with single digit numbers can be used to calculate complements to 100 and add and subtract across 100



$7 + 3 = 10$ $70 + 30 = 100$
 $10 - 3 = 7$ $100 - 30 = 70$



- 'I know that seven plus five is equal to twelve.'
- 'So seven tens plus five tens is equal to twelve tens.'
- 'Seventy plus fifty is equal to one hundred and twenty.'

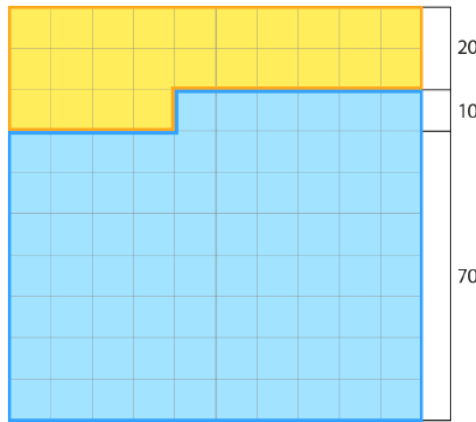
$$7 + 5 = 12$$

$$7 \text{ tens} + 5 \text{ tens} = 12 \text{ tens}$$

$$70 + 50 = 120$$

Partitioning

Addition of complements to 100 can be done by partitioning both addends (the numbers being added together) into tens and ones

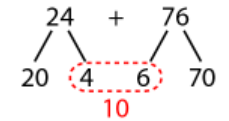


$$24 + 76$$

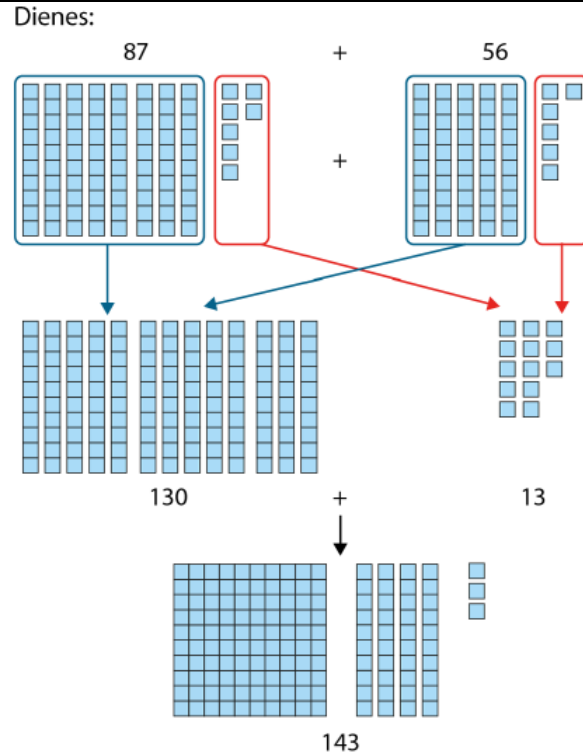
$$20 + 70 = 90$$

$$4 + 6 = 10$$

$$90 + 10 = 100$$



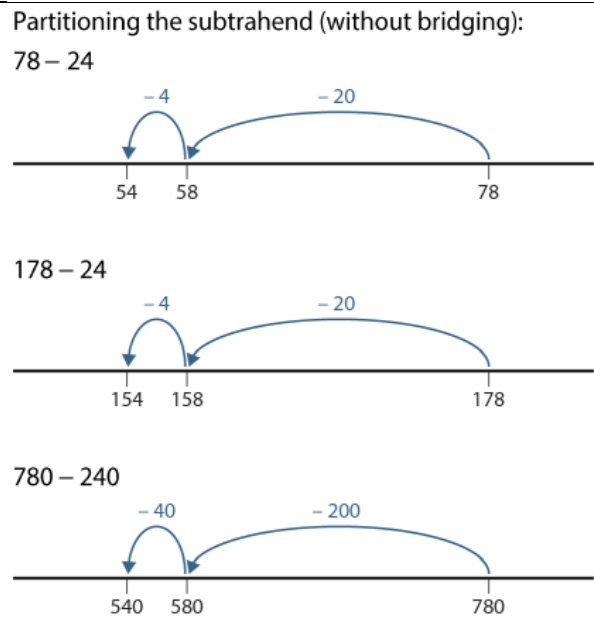
Partitioning can be used to add both two-digit and three-digit numbers



Jottings:

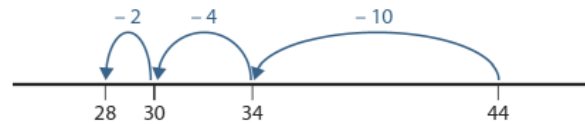
$$\begin{array}{r} 87 \\ 80 \quad 7 \end{array} + \begin{array}{r} 56 \\ 50 \quad 6 \end{array} = 130 + 13 = 143$$

Partitioning the subtrahend (the number being taken away) can be used when subtracting. This method can be used either without bridging or with bridging.

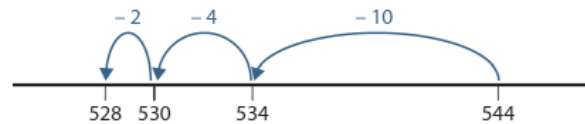


Partitioning the subtrahend (with bridging):

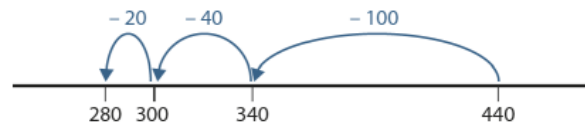
$$44 - 16$$



$$544 - 16$$



$$440 - 160$$

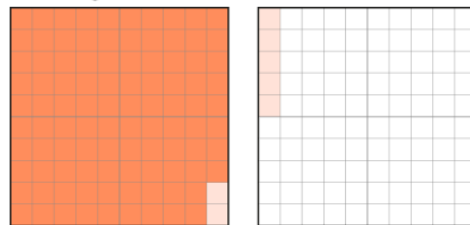


Making 100

Bridging 100 (adding or subtracting across 100) can be done by first making 100

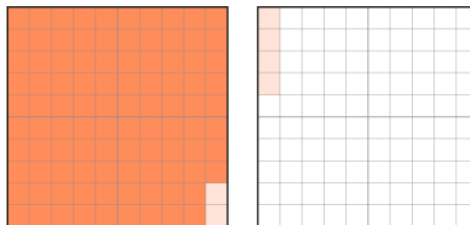
$$98 + 7$$

Hundred grids:

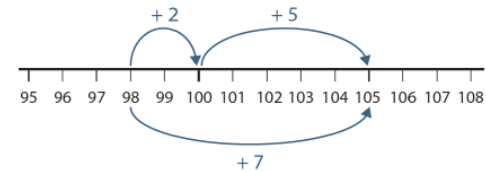


$$104 - 6$$

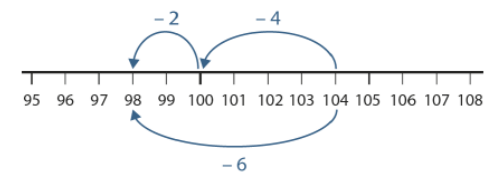
Hundred grids:



Number line:



Number line:



Jotting and equations:

$$98 + 7 = 105$$

$$\begin{aligned} 98 + 7 &= 98 + 2 + 5 \\ &= 100 + 5 \\ &= 105 \end{aligned}$$

Jotting and equations:

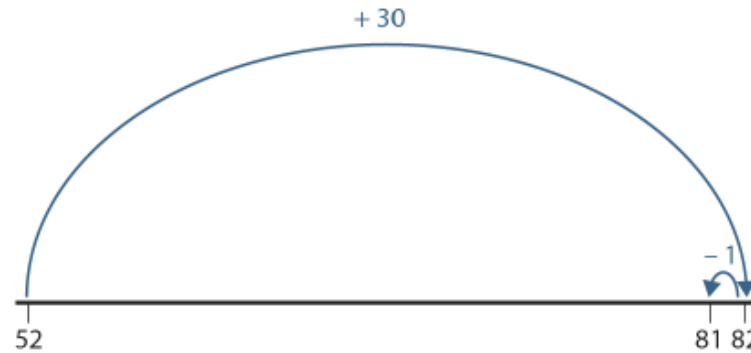
$$104 - 6 = 98$$

$$\begin{aligned} 104 - 6 &= 104 - 4 - 2 \\ &= 100 - 2 \\ &= 98 \end{aligned}$$

Adjusting

Adjusting is a more efficient addition strategy than partitioning when one of the numbers involved is close to a multiple of 10 or 100 (e.g. 49 is close to 50).

In the example given, 30 is added rather than 29 as it is a simpler calculation. 1 is then subtracted to adjust for the extra 1 that was added.

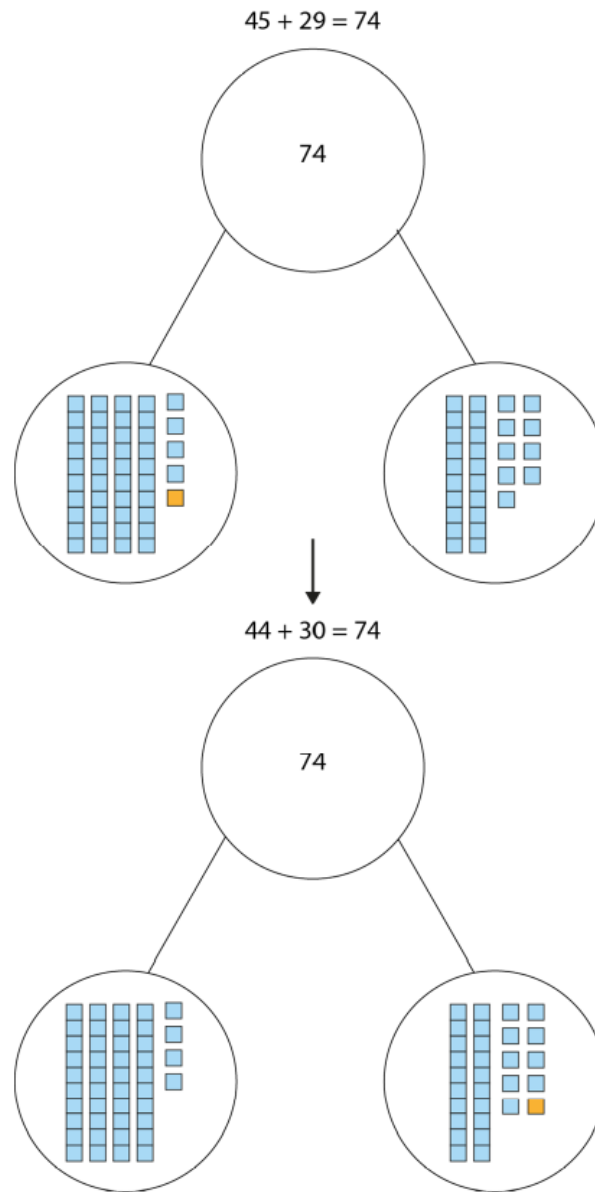


$$\begin{aligned} 52 + 29 &= 52 + 30 - 1 \\ &= 82 - 1 \\ &= 81 \end{aligned}$$

Redistributing

In the redistribution strategy an addition calculation is made simpler by increasing one addend and decreasing the other by the same amount.

There are similarities between the redistributing strategy and the adjusting strategy. However, with redistribution, the total remains the same at all times whereas with adjusting the total amount is increased to simplify the calculation and then decreased again.

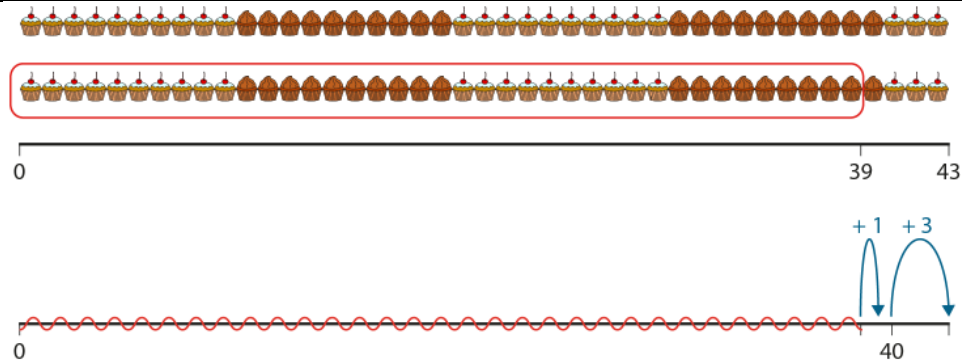


$$45 + 29 = 44 + 30 = 74$$

Finding the difference (adding on)

In this strategy, start with the subtrahend and add on to reach the minuend. The amount needed to be added will be the difference and the answer to the calculation.

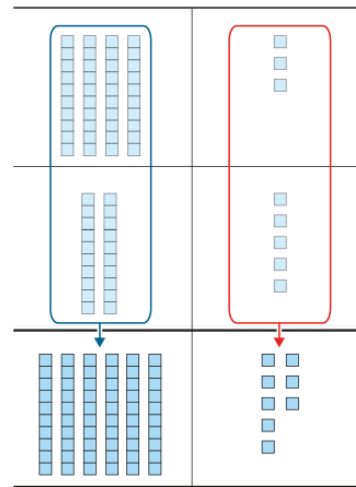
This strategy is particularly useful when the minuend and subtrahend are close together (e.g. $43 - 39$)



$$43 - 39 = 4$$

Column Addition


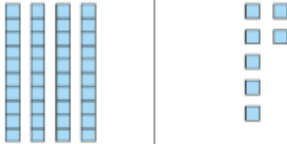
Column addition is the formal written method for addition taught and used throughout KS2 and beyond for times when an efficient mental method is either not known or cannot be used to a suitable degree of accuracy.



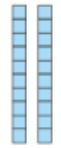
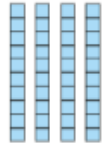

$$\begin{array}{r} 43 \\ + 25 \\ \hline 68 \end{array}$$

When the total of any column is 10 or greater, we must regroup. In the example shown, this involves exchanging 10 ones within the number 12 for 1 ten. This leaves 2 in the ones column and 1 ten below the tens column to be added when the tens are added.

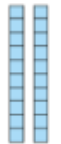
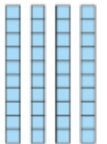

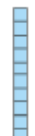
Step 1

	
	$\begin{array}{r} 25 \\ + 47 \\ \hline \end{array}$

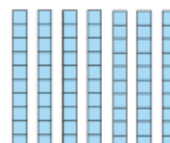

Step 2

	
	$\begin{array}{r} 25 \\ + 47 \\ \hline \end{array}$
	

Step 3

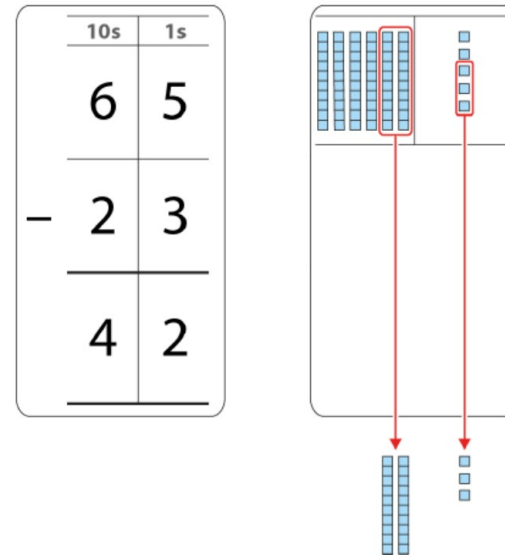
	
	$\begin{array}{r} 25 \\ + 47 \\ \hline 2 \\ \hline 1 \end{array}$
	
	

Step 4

	$\begin{array}{r} 25 \\ + 47 \\ \hline 72 \\ \hline 1 \end{array}$
	

Column Subtraction

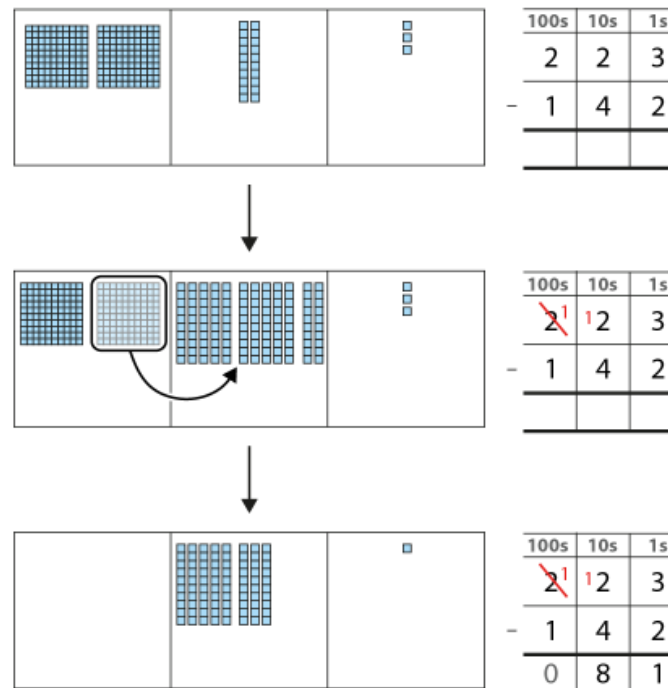
Similar to column addition, column subtraction is the formal written method for subtraction taught and used throughout KS2 and beyond for times when an efficient mental method is either not known or cannot be used to a suitable degree of accuracy.



When the subtrahend (the number on the second row) in any column is greater than the minuend above it (the number at the top), we must regroup.

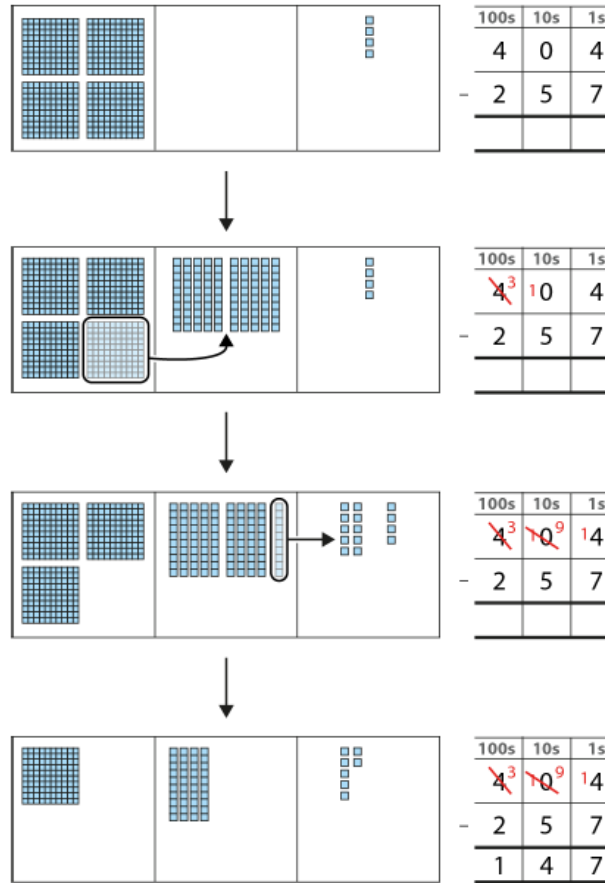
In the example shown (where 4 tens cannot be taken away from 2 tens), this involves exchanging 1 of the hundreds for 10 tens leaving 1 hundred remaining in the hundreds column and combining the exchanged 10 tens with the existing 2 tens to give 12 tens in the tens column. This allows 4 tens to be taken away from the 12 tens.

$$223 - 142$$



Regrouping can sometimes require working through a column with zero because the zero shows there is nothing to be exchanged.

In this situation, as shown in the example, regrouping can be done by exchanging from the next column to the left (the hundreds in this case). The regrouping must first be done into the column with zero (so exchanging 1 hundred into ten tens) which can then lead to regrouping into the column where the initial subtraction wasn't possible (so exchanging 1 of the previously exchanged tens into 10 ones).

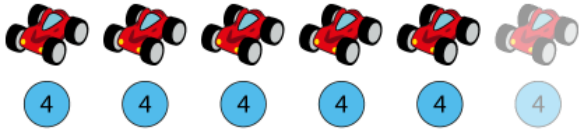
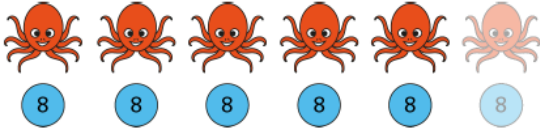


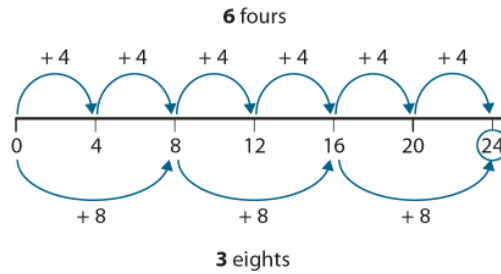
Year 3 – multiplication and division

Within Year 3, the children continue to develop their times table knowledge by recalling the 5x and 2x tables learnt in KS1 and learning their times table number facts for the 4x and 8x tables. The 2x, 4x and 8x tables are taught in this sequence to reinforce the doubling relationship between them. Once the 2x, 4x and 8x tables are secure, the children then learn the 3x, 6x and 9x tables and the relationship between them.

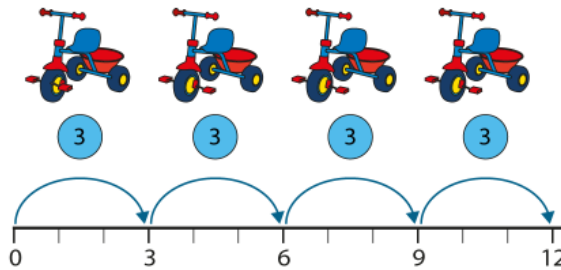
While a formal written method for multiplication and division is not taught in Year 3, the acquisition of times table knowledge is essential for the children to be ready to learn these in Year 4.

Number Facts

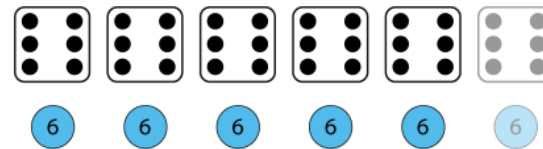
	Concrete and Pictorial Representations	Abstract																																																												
Times table number facts: 5x and 2x table (This is a recap of KS1 learning)																																																														
Times table number facts: 2x, 4x and 8x table and the relationship between them	<div style="display: flex; justify-content: space-around; align-items: center;">  <table border="1" data-bbox="1283 671 1563 1029"> <thead> <tr> <th>Number of cars</th> <th>Total number of wheels</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>4</td></tr> <tr><td>2</td><td>8</td></tr> <tr><td>3</td><td>12</td></tr> <tr><td>4</td><td>16</td></tr> <tr><td>5</td><td>20</td></tr> <tr><td>6</td><td>24</td></tr> </tbody> </table> </div> <div style="display: flex; justify-content: space-around; align-items: center;">  <table border="1" data-bbox="1279 1114 1556 1445"> <thead> <tr> <th>Number of octopuses</th> <th>Total number of tentacles</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>8</td></tr> <tr><td>2</td><td>16</td></tr> <tr><td>3</td><td>24</td></tr> <tr><td>4</td><td>32</td></tr> <tr><td>5</td><td>40</td></tr> <tr><td>6</td><td>48</td></tr> </tbody> </table> </div>	Number of cars	Total number of wheels	0	0	1	4	2	8	3	12	4	16	5	20	6	24	Number of octopuses	Total number of tentacles	0	0	1	8	2	16	3	24	4	32	5	40	6	48	<table border="1" data-bbox="1648 671 2107 919"> <tbody> <tr><td>$0 \times 4 = 0$</td><td>$4 \times 0 = 0$</td></tr> <tr><td>$1 \times 4 = 4$</td><td>$4 \times 1 = 4$</td></tr> <tr><td>$2 \times 4 = 8$</td><td>$4 \times 2 = 8$</td></tr> <tr><td>$3 \times 4 = 12$</td><td>$4 \times 3 = 12$</td></tr> <tr><td>$4 \times 4 = 16$</td><td>$4 \times 4 = 16$</td></tr> <tr><td>$5 \times 4 = 20$</td><td>$4 \times 5 = 20$</td></tr> <tr><td>$6 \times 4 = 24$</td><td>$4 \times 6 = 24$</td></tr> </tbody> </table> <table border="1" data-bbox="1653 1110 2101 1353"> <tbody> <tr><td>$0 \times 8 = 0$</td><td>$8 \times 0 = 0$</td></tr> <tr><td>$1 \times 8 = 8$</td><td>$8 \times 1 = 8$</td></tr> <tr><td>$2 \times 8 = 16$</td><td>$8 \times 2 = 16$</td></tr> <tr><td>$3 \times 8 = 24$</td><td>$8 \times 3 = 24$</td></tr> <tr><td>$4 \times 8 = 32$</td><td>$8 \times 4 = 32$</td></tr> <tr><td>$5 \times 8 = 40$</td><td>$8 \times 5 = 40$</td></tr> <tr><td>$6 \times 8 = 48$</td><td>$8 \times 6 = 48$</td></tr> </tbody> </table>	$0 \times 4 = 0$	$4 \times 0 = 0$	$1 \times 4 = 4$	$4 \times 1 = 4$	$2 \times 4 = 8$	$4 \times 2 = 8$	$3 \times 4 = 12$	$4 \times 3 = 12$	$4 \times 4 = 16$	$4 \times 4 = 16$	$5 \times 4 = 20$	$4 \times 5 = 20$	$6 \times 4 = 24$	$4 \times 6 = 24$	$0 \times 8 = 0$	$8 \times 0 = 0$	$1 \times 8 = 8$	$8 \times 1 = 8$	$2 \times 8 = 16$	$8 \times 2 = 16$	$3 \times 8 = 24$	$8 \times 3 = 24$	$4 \times 8 = 32$	$8 \times 4 = 32$	$5 \times 8 = 40$	$8 \times 5 = 40$	$6 \times 8 = 48$	$8 \times 6 = 48$
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Times table number facts:
3x, 6x and 9x table and the
relationship between them



Number of tricycles	Total number of wheels
0	0
1	3
2	6
3	9
4	12
5	15
6	18

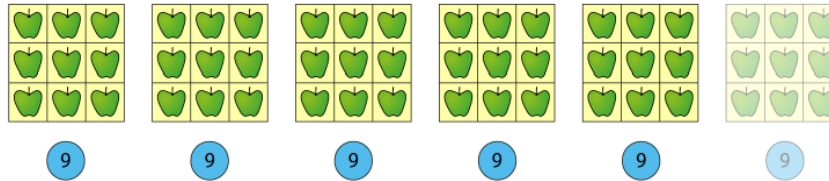


Number of six-value dice	Total number of dots
0	0
1	6
2	12
3	18
4	24
5	30
6	36

Number	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Counting in 3s	✓			✓			✓			✓			✓			✓			✓			✓			✓
Counting in 6s	✓						✓						✓						✓						✓

$0 \times 3 = 0$	$3 \times 0 = 0$
$1 \times 3 = 3$	$3 \times 1 = 3$
$2 \times 3 = 6$	$3 \times 2 = 6$
$3 \times 3 = 9$	$3 \times 3 = 9$
$4 \times 3 = 12$	$3 \times 4 = 12$
$5 \times 3 = 15$	$3 \times 5 = 15$
$6 \times 3 = 18$	$3 \times 6 = 18$

$0 \times 6 = 0$	$0 \times 6 = 0$
$1 \times 6 = 6$	$1 \times 6 = 6$
$2 \times 6 = 12$	$2 \times 6 = 12$
$3 \times 6 = 18$	$3 \times 6 = 18$
$4 \times 6 = 24$	$4 \times 6 = 24$
$5 \times 6 = 30$	$5 \times 6 = 30$
$6 \times 6 = 36$	$6 \times 6 = 36$



Number of boxes of 9 apples	Total number of apples
0	0
1	9
2	18
3	27
4	36
5	45
6	54

$$0 \times 9 = 0$$

$$1 \times 9 = 9$$

$$2 \times 9 = 18$$

$$3 \times 9 = 27$$

$$4 \times 9 = 36$$

$$5 \times 9 = 45$$

$$6 \times 9 = 54$$

$$9 \times 0 = 0$$

$$9 \times 1 = 9$$

$$9 \times 2 = 18$$

$$9 \times 3 = 27$$

$$9 \times 4 = 36$$

$$9 \times 5 = 45$$

$$9 \times 6 = 54$$

Application of Number Facts

Concrete and Pictorial Representations

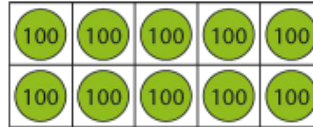
Abstract

Number facts to 1,000

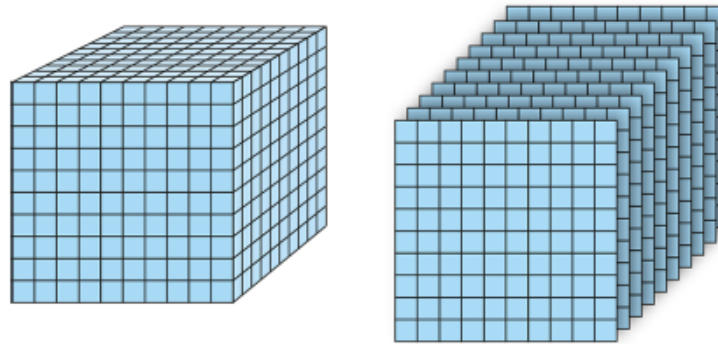
(These are explored both as additive and multiplicative equations and applied within the range of strategies listed below)

Representing ten hundreds in 1,000:

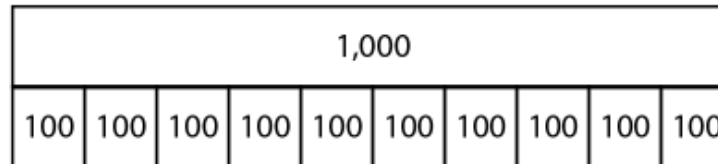
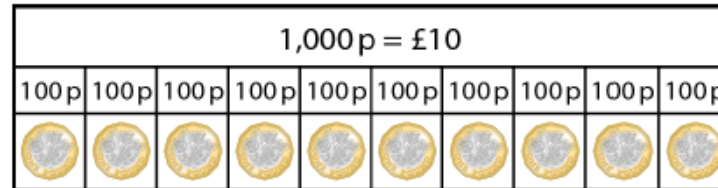
- Tens frame and 100 place-value counters



- Dienes



- Coins



- Additive and multiplicative equations

$$1,000 = 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100$$

$$1,000 = 10 \times 100 \quad 1,000 = 100 \times 10$$

$$1,000 \div 100 = 10 \quad 1,000 \div 10 = 100$$

1,000	
500	500

1,000			
250	250	250	250

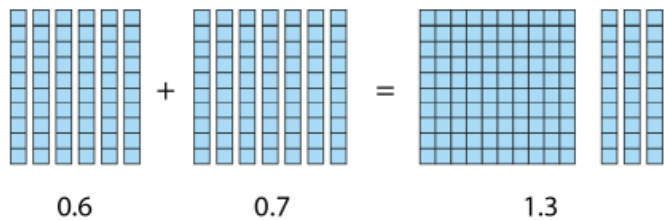
1,000				
200	200	200	200	200

Mental Strategies with 4-digit numbers and decimals

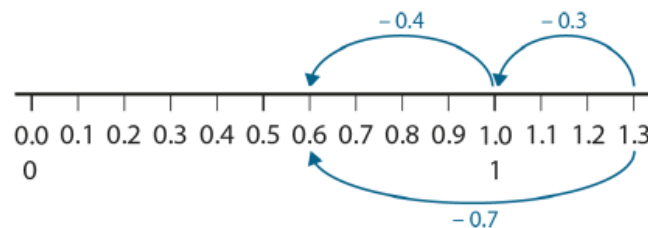
Each of the mental strategies taught in Year 3 are applied and explored within numbers in the thousands. This gives a chance for the children to be reminded of those strategies and gain familiarity in using them with increasing confidence while working with 4-digit numbers. These are further developed and extended once the children have learnt about decimals (tenths, hundredths and thousands). Examples of these strategies being used for decimals can be seen below.

See the Year 3 section above for an explanation and example of each mental strategy covered.

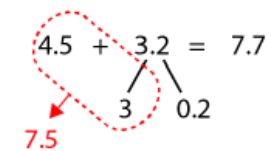
Dienes – example addition calculation:



Number line – example subtraction calculation:



Applying known strategies – partitioning numbers with tenths:



Column Addition and subtraction

The column addition and subtraction algorithms taught in Y3 are extended and built upon in Y4 to include addition and subtraction of 4-digit numbers and decimals.

For an explanation of these methods and how they are introduced, see the Y3 section above.

$$\begin{array}{r} 13.2 \\ + 5.7 \\ \hline \end{array} \qquad \begin{array}{r} 36.5 \\ - 2.3 \\ \hline \end{array}$$

Year 4 – multiplication and division

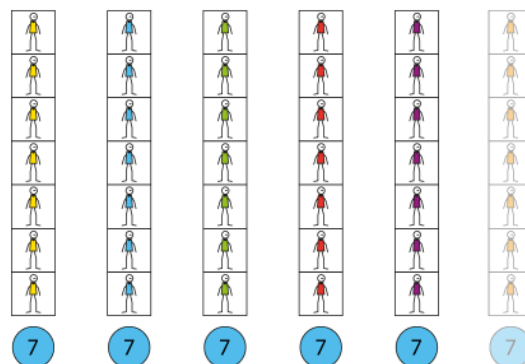
Initially, the times table facts taught in Y3 are recapped to ensure these are secure before moving on to learn the 7x, 11x and 12x tables. This prepares them for a range of mental and written strategies for multiplication and division as well as for the statutory Multiplication Tables Check (a test given to all children in Year 4 to assess fluency of times tables recall).

Children in Year 4 are also taught a formal written method for multiplication and division: short multiplication and short division.

Number Facts

Times table number facts:
7x, 11x and 12x tables

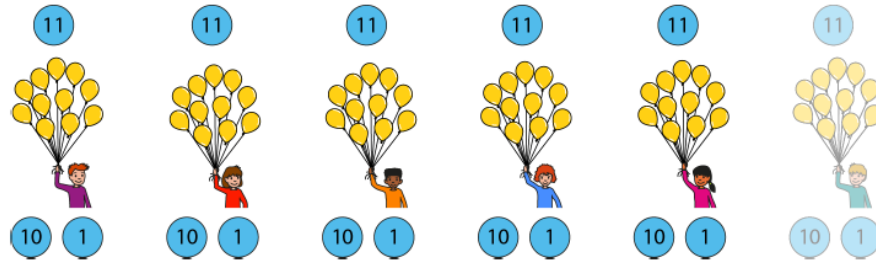
Concrete and Pictorial Representations



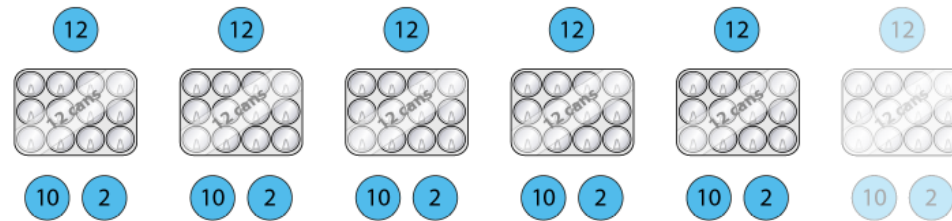
Number of netball teams	Total number of players
0	0
1	7
2	14
3	21
4	28
5	35
6	42

Abstract

$0 \times 7 = 0$	$7 \times 0 = 0$
$1 \times 7 = 7$	$7 \times 1 = 7$
$2 \times 7 = 14$	$7 \times 2 = 14$
$3 \times 7 = 21$	$7 \times 3 = 21$
$4 \times 7 = 28$	$7 \times 4 = 28$
$5 \times 7 = 35$	$7 \times 5 = 35$
$6 \times 7 = 42$	$7 \times 6 = 42$



Number of bunches of balloons	$\times 10$	$\times 1$	Total number of balloons ($\times 11$)
0	0	0	0
1	10	1	11
2	20	2	22
3	30	3	33
4	40	4	44
5	50	5	55
6	60	6	66



$0 \times 11 = 0$	$11 \times 0 = 0$
$1 \times 11 = 11$	$11 \times 1 = 11$
$2 \times 11 = 22$	$11 \times 2 = 22$
$3 \times 11 = 33$	$11 \times 3 = 33$
$4 \times 11 = 44$	$11 \times 4 = 44$
$5 \times 11 = 55$	$11 \times 5 = 55$
$6 \times 11 = 66$	$11 \times 6 = 66$

$0 \times 12 = 0$	$12 \times 0 = 0$
$1 \times 12 = 12$	$12 \times 1 = 12$
$2 \times 12 = 24$	$12 \times 2 = 24$
$3 \times 12 = 36$	$12 \times 3 = 36$
$4 \times 12 = 48$	$12 \times 4 = 48$
$5 \times 12 = 60$	$12 \times 5 = 60$
$6 \times 12 = 72$	$12 \times 6 = 72$

Number of packs of cans	$\times 10$	$\times 2$	Total number of cans ($\times 12$)
0	0	0	0
1	10	2	12
2	20	4	24
3	30	6	36
4	40	8	48
5	50	10	60
6	60	12	72

Multiplying and dividing by 10 and 100

Concrete and Pictorial Representations

Abstract

The mental strategy for multiplying and dividing by 10 and 100 involves recognising the patterns within place value columns.

For example, when a number is multiplied by 10, all of the digits move one place to the left (the number in the 1s moving to the 10s). This means that all of the digits will stay in the same order but will have a place holder in the 1s column).

Gattegno chart:

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

$\times 10$ (indicated by an arrow pointing left from the bottom row to the top row)
 $\div 10$ (indicated by an arrow pointing right from the top row to the bottom row)

1,000s	100s	10s	1s
			●
		●	
	●		
●			

ten times the size ten times the size ten times the size

1,000s	100s	10s	1s
			1
		1	
	1		
1			

ten times the size ten times the size ten times the size

Step 1 – move each of the digits one place to the left

1,000s	100s	10s	1s
		1	2
	1	2	

↓ × 10 Think of '12' and make it ten times the size.



Step 2 – write a '0' in the ones place

1,000s	100s	10s	1s
		1	2
	1	2	0

↓ × 10 Think of '12' and make it ten times the size.



Step 1 – move each of the digits two places to the left

1,000s	100s	10s	1s
		1	5
1	5		

↓ × 100 Think of '15' and make it 100 times the size.



Step 2 – introduce zeros in the tens and ones places

1,000s	100s	10s	1s
		1	5
1	5	0	0

↓ × 100 Think of '15' and make it 100 times the size.



Ratio chart:

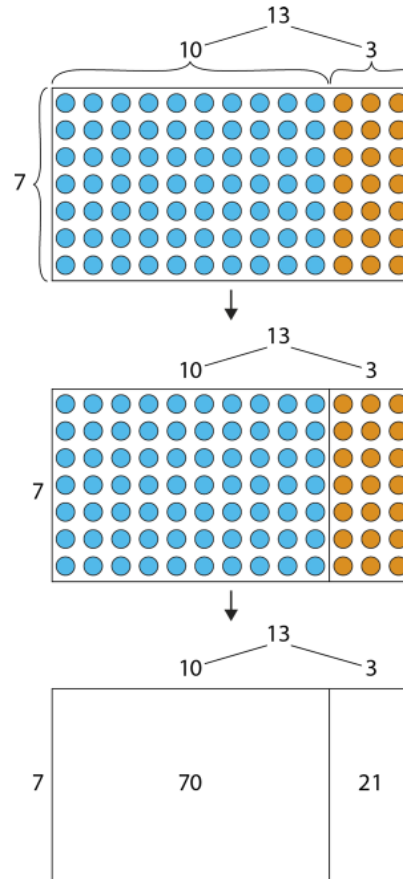
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
÷ 10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
÷ 10	0	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500

Partitioning for multiplication

Concrete and Pictorial Representations

By applying knowledge of the distributive law (which means that if you split a number and multiply the split parts separately and add the separate answers together, you get the same answer as would get if you had multiplied the original number. The example here demonstrates this with 13×7 . The 13 can be partitioned into a 10 and 3 with each of the partitions multiplied by 7. The products for those calculations can be added to find the final answer.

This strategy can be used both in written form and mentally, depending on the numbers involved and the strength of number fact knowledge.



Abstract

$$\begin{aligned} 13 \times 7 &= 10 \times 7 + 3 \times 7 & 7 \times 13 &= 7 \times 10 + 7 \times 3 \\ &= 70 + 21 & &= 70 + 21 \\ &= 91 & &= 91 \end{aligned}$$

Partitioning for division

Concrete and Pictorial Representations

Abstract

Similar to multiplication, partitioning can be used to divide 2-digit numbers by single digit numbers.

The example here shows how this can be done by splitting 84 into 8 tens and 4 ones which can each be divided by 4 and combined to reach the final answer.

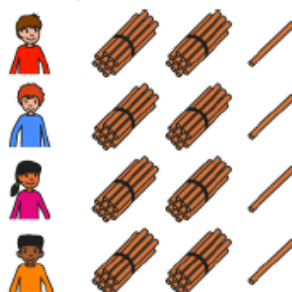
This is developed further later in KS2 when non-standard partitioning (splitting a number in a way other than by place value) can be used for efficient mental calculation. An example of this would be to solve $56 \div 4$ by partitioning 56 into known multiples of 4 such as 40 and 16. Knowing that:
 $40 \div 4 = 10$
 and
 $16 \div 4 = 4$
 can allow the answer of 14 to efficiently calculated without the need for a written method.

'Eighty-four sticks are shared equally between four children. How many sticks does each child get?'

$$84 \div 4 = ?$$



Example solution reached by partitioning into tens and ones and dividing these separately. The quotients (answer for a division calculation) are then added.



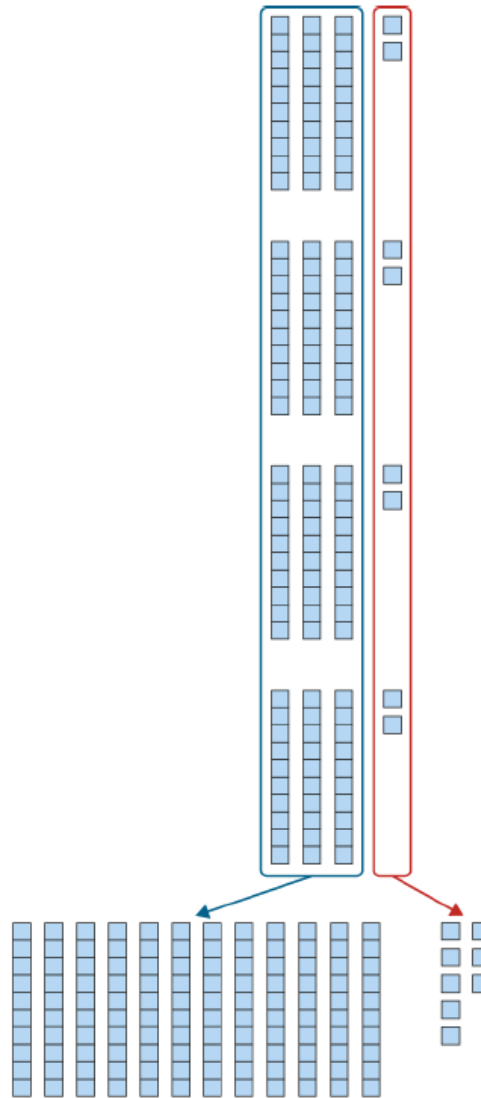
8 tens	÷	4	=	2 tens
4 ones	÷	4	=	1 one
84	÷	4	=	21

Short multiplication

Short multiplication is a written method for multiplication taught as way to multiply a 2-digit number by a single digit number (such as 17×6). This is later extended to multiply 3 and 4-digit numbers by a single digit in Years 5 and 6.

This method is initially introduced using physical resources such as dienes to represent what happens within the multiplication before moving to the written layout. It builds on their understanding of partitioning for multiplication explained above.

Concrete and Pictorial Representations



Abstract

$$32 \times 4 = 30 \times 4 + 2 \times 4 \\ = 120 + 8$$

- *Three-tens-and-two-ones multiplied by four is equal to three tens multiplied by four and two ones multiplied by four.*

$$3 \text{ tens} \times 4 = 12 \text{ tens}$$

$$2 \text{ ones} \times 4 = 8 \text{ ones}$$

Example 1 – compact layout *with* place-value headings:

$$\begin{array}{r|l} 10\text{s} & 1\text{s} \\ \hline 3 & 2 \\ \times & 3 \\ \hline 9 & 6 \end{array}$$

- 3×2 ones = 6 ones
'Write "6" in the ones column.'
- 3×3 tens = 9 tens
'Write "9" in the tens column.'

Example 2 – compact layout *without* place-value headings:

$$\begin{array}{r} 2 \ 1 \\ \times \ 4 \\ \hline 8 \ 4 \end{array}$$

- 4×1 one = 4 ones
'Write "4" in the ones column.'
- 4×2 tens = 8 tens
'Write "8" in the tens column.'

When the product of one column is greater than 9, regrouping needs to be done.

In this example, 3×4 gives the product of 12 ones which cannot fit in the one's column. Therefore, 10 ones are exchanged for 1 ten which is written below the tens column and the 2 is written in the ones column.

Example 1 – compact layout *with* place-value headings:

Step 1 – write the factors:

$$\begin{array}{r|l} 10\text{s} & 1\text{s} \\ \hline & 2 \quad 4 \\ \times & 3 \\ \hline \end{array}$$

Step 2 – multiply the single-digit number by the ones and regroup:

$$\begin{array}{r|l} 10\text{s} & 1\text{s} \\ \hline & 2 \quad 4 \\ \times & 3 \\ \hline & 2 \\ \hline 1 & \end{array}$$

$3 \times 4 \text{ ones} = 12 \text{ ones}$
 $= 1 \text{ ten} + 2 \text{ ones}$

'Write "1" below the tens column and "2" in the ones column.'

Step 3 – multiply the single-digit number by the tens and add the tens from regrouping:

$$\begin{array}{r|l} 10\text{s} & 1\text{s} \\ \hline & 2 \quad 4 \\ \times & 3 \\ \hline & 7 \quad 2 \\ \hline 1 & \end{array}$$

$3 \times 2 \text{ tens} = 6 \text{ tens}$

$6 \text{ tens} + 1 \text{ ten} = 7 \text{ tens}$
'Write "7" in the tens column.'

Example 2 – compact layout *without* place-value headings:

$$\begin{array}{r} 1 \quad 8 \\ \times \quad 5 \\ \hline 9 \quad 0 \\ \hline 4 \end{array}$$

- $5 \times 8 \text{ ones} = 40 \text{ ones}$; $40 \text{ ones} = 4 \text{ tens}$ and 0 ones
'Write "4" below the tens column and "0" in the ones column.'
- $5 \times 1 \text{ ten} = 5 \text{ tens}$
 $5 \text{ tens} + 4 \text{ tens} = 9 \text{ tens}$
'Write "9" in the tens column.'

Short division

Concrete and Pictorial Representations

Abstract

Short division is a written method for division taught as way to divide a 2-digit number by a single digit number (such as $84 \div 4$). This is later extended to divide by two-digit numbers in Years 5 and 6 but remains the most efficient written method for dividing by a single digit.

This method is initially introduced using physical resources such as dienes and pictorial representations to explore the method before moving to the written layout. It builds on their understanding of partitioning for division explained above.

Step 1 – write the divisor and dividend



10s 1s

$$\begin{array}{r} 4 \overline{) 84} \end{array}$$

'Eighty-four divided by four.'

Step 2 – sharing the tens



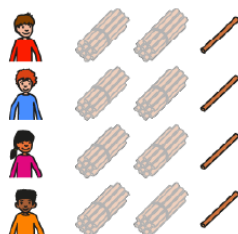
10s 1s

$$\begin{array}{r} 2 \\ 4 \overline{) 84} \end{array}$$

$8 \text{ tens} \div 4 = 2 \text{ tens}$

'Eight tens divided by four is equal to two tens.'

Step 3 – sharing the ones



10s 1s

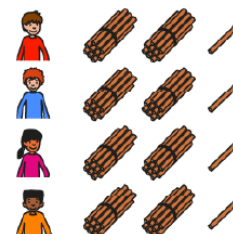
$$\begin{array}{r} 2 1 \\ 4 \overline{) 84} \end{array}$$

$8 \text{ tens} \div 4 = 2 \text{ tens}$

$4 \text{ ones} \div 4 = 1 \text{ one}$

'Four ones divided by four is equal to one one.'

Summary



10s 1s

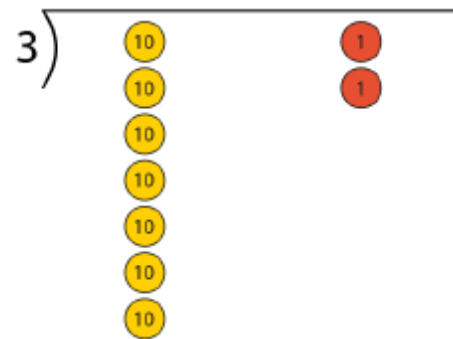
$$\begin{array}{r} 2 1 \\ 4 \overline{) 84} \end{array}$$

'Each child gets twenty-one sticks.'

When the division within a place value column leaves a remainder, exchanging is used.

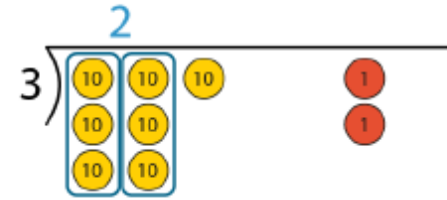
In this example, when the 7 tens are divided by 3 there is a remainder of 1 ten. This is then exchanged for 10 ones giving a total of 12 ones to be divided next.

Step 1 – write the divisor and dividend



$$3 \overline{) 72}$$

Step 2 – sharing the tens...



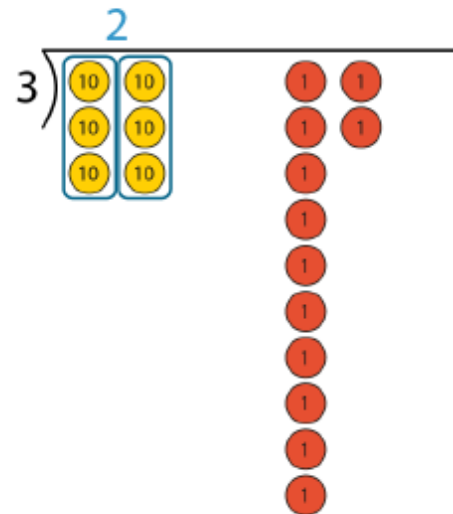
$$3 \overline{) 72} \begin{array}{r} 2 \\ \hline \end{array}$$

'Seventy-two divided by three.'

7 tens \div 3 = 2 tens r 1 ten

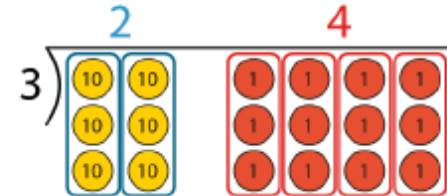
'Write "2" in the tens column...'

Step 3 – ...and exchanging



$$3 \overline{) 72} \begin{array}{r} 2 \\ \hline 7 \ 12 \end{array}$$

Step 4 – sharing the ones



$$3 \overline{) 72} \begin{array}{r} 2 \ 4 \\ \hline 7 \ 12 \end{array}$$

1 ten = 10 ones

'...and write "1" to the left of the ones digit of the dividend to make twelve ones.'

12 ones \div 3 = 4 ones

'Write "4" in the ones column.'

Year 5 – addition and subtraction

Application of Number Facts

Concrete and Pictorial Representations

Abstract

Number facts to 1,000,000
 (These are explored both as additive and multiplicative equations and applied within the range of strategies listed below)

10,000									
1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

100,000									
10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000

10,000	
5,000	5,000

$$10,000 = 5,000 \times 2$$

$$10,000 \div 2 = 5,000$$

10,000			
2,500	2,500	2,500	2,500

$$10,000 = 2,500 \times 4$$

$$10,000 \div 4 = 2,500$$

10,000				
2,000	2,000	2,000	2,000	2,000

$$10,000 = 2,000 \times 5$$

$$10,000 \div 5 = 2,000$$

100,000	
50,000	50,000

$$100,000 = 50,000 \times 2$$

$$100,000 \div 2 = 50,000$$

100,000			
25,000	25,000	25,000	25,000

$$100,000 = 25,000 \times 4$$

$$100,000 \div 4 = 25,000$$

100,000				
20,000	20,000	20,000	20,000	20,000

$$100,000 = 20,000 \times 5$$

$$100,000 \div 5 = 20,000$$

Mental Strategies with 5 and 6-digit numbers

Each of the mental strategies taught in Year 3 and used in Year 4 with 4-digit numbers are applied and explored within numbers in the tens of thousands and hundreds of thousands. This gives a chance for the children to be reminded of those strategies and gain familiarity in using them with increasing confidence while working with 5 and 6-digit numbers.

See the Year 3 section above for an explanation and example of each mental strategy covered.

Column Addition and subtraction

The column addition and subtraction algorithms taught in Y3 and extended to include 4-digit numbers in Y4 are further extended into the tens of thousands and hundreds of thousands. Examples of these can be seen below.

For an explanation of these methods and how they are introduced, see the Y3 section above.

Column addition and subtraction:

- With place-value headings

Thousands			Ones		
100s	10s	1s	100s	10s	1s
3	6	5	0	0	0
+	2	1	4	0	0
5	7	9	0	0	0

- Without place-value headings

$$\begin{array}{r} 3\ 6\ 5,000 \\ +\ 2\ 1\ 4,000 \\ \hline 5\ 7\ 9,000 \end{array}$$

By the time they enter Year 5, children are expected to be able to confidently recall their times tables up to 12x12. This knowledge forms the foundation for learning more advanced methods and working with both larger numbers and with decimals.

Children in Year 5 are also taught a formal written method for multiplying and dividing by two digit numbers: long multiplication and long division.

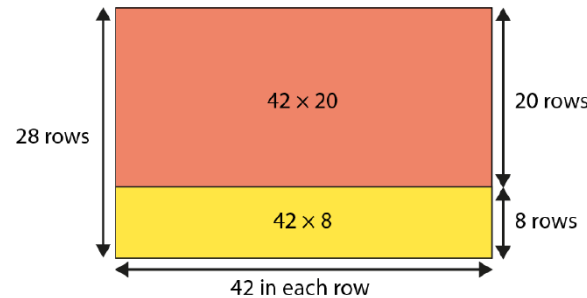
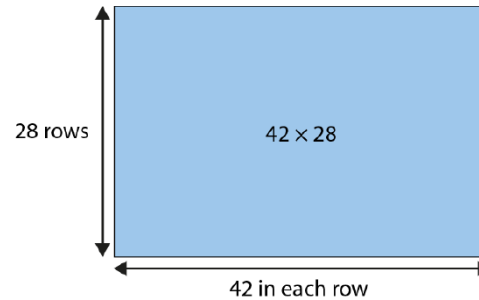
Long Multiplication

Long multiplication is introduced by first using short multiplication by both the ones and tens separately. These answers (known as partial products) can then be added to find the product of the complete multiplication.

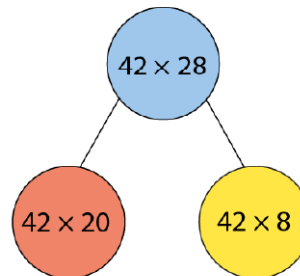
This introduces the children to the fact that multiplication by a 2 digit number can be done by first multiplying by the ones, then the tens and these products then added.

Concrete and Pictorial Representations

Area model/grid:



Part-part-whole model:



Abstract

Short multiplication and combining partial products:

$$\begin{array}{r} 42 \\ \times 8 \\ \hline 336 \\ 1 \end{array} \quad \begin{array}{r} 42 \\ \times 20 \\ \hline 840 \end{array}$$

$$\begin{array}{r} 840 \\ + 336 \\ \hline 1176 \end{array}$$

The separate short multiplication and addition steps are then combined within the long multiplication algorithm. To the right is shown the expanded layout. This most clearly shows each of the steps within the method. Children will quickly move from this expanded layout shown below once they are secure with the method.

Multiplication algorithm – expanded layout:

Step 1 – write the factors

	100s	10s	1s
		3	1
×		2	4

Step 2 – multiply the ones digit by the ones digit

	100s	10s	1s
		3	1
×		2	4
			4

$4 \times 1 \text{ one} = 4 \text{ ones}$

Step 3 – multiply the tens digit by the ones digit and regroup

	100s	10s	1s
		3	1
×		2	4
	1	2	4

$4 \times 3 \text{ tens} = 12 \text{ tens}$
 $= 1 \text{ hundred} + 2 \text{ tens}$

Step 4 – place a zero to show that it's ten times the size

	100s	10s	1s
		3	1
×		2	4
	1	2	4
			0

Step 5 – multiply the ones digit by the tens digit

	100s	10s	1s
		3	1
×		2	4
	1	2	4
		2	0

$2 \text{ tens} \times 1 \text{ one} = 2 \text{ tens}$

Step 6 – multiply the tens digit by the tens digit

	100s	10s	1s
		3	1
×		2	4
	1	2	4
	6	2	0

$2 \text{ tens} \times 3 \text{ tens} = 6 \text{ hundreds}$

Step 7 – add the partial products

	100s	10s	1s
		3	1
×		2	4
	1	2	4
	6	2	0
	7	4	4

31×4
 31×20

Multiplication algorithm – expanded layout:

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	31×4
	6	2	0	31×20
	7	4	4	

Multiplication algorithm – compact layout:

	3	1	
×	2	4	
	1	2	4
	6	2	0
	7	4	4

Long Division

Concrete and Pictorial Representations

Abstract

The method for long division is taught to provide a formal written method when dividing by two-digit numbers that cannot be calculated mentally. Before attempting the written method, a ratio chart (frequently referred to as a 'What I Know' box of W.I.K.) must be created. This is a starter list of times tables for the divisor. It is not a complete list; it contains those that can be calculated quickly and acts as a starting point to work out the others if needed.

The 2x, 4x, and 8x can be found by doubling, doubling and doubling again. The 10x can be found by using knowledge of place value and the 5x can be found by halving the 10x.

The initial layout for long division is very similar to short division with the key difference being that the subtraction step is recorded, rather than being held mentally. This is to avoid errors which would likely occur if all of the steps of the division were held mentally.

Ratio chart:

	× 31
1	31
2	62
3	
4	124
5	155
6	
7	
8	248
9	
10	310

Step 1 – write the divisor, frame and dividend

$$31 \overline{) 434}$$

Step 2 – divide the hundreds

$$\begin{array}{r} 0 \\ 31 \overline{) 434} \end{array}$$

4 hundreds ÷ 31 = 0 hundreds r 4 hundreds

- 'Write "0" in the hundreds column of the answer line.'

Step 3 – exchange hundreds for tens, combine with the existing tens and divide...

$$\begin{array}{r} 0 \quad 1 \\ 31 \overline{) 434} \\ \underline{31} \\ 31 \end{array} \quad (1 \text{ ten} \times 31 = 31 \text{ tens})$$

4 hundreds = 40 tens

40 tens + 3 tens = 43 tens

43 tens ÷ 31 = 1 ten and a remainder

- 'Write "1" in the tens column of the answer line and write "31" underneath the "43".'

Step 4 – subtract to find the remainder

$$\begin{array}{r} 0 \ 1 \\ 31 \overline{) 4 \ 3 \ 4} \\ \underline{3 \ 1} \quad (1 \text{ ten} \times 31 = 31 \text{ tens}) \\ 1 \ 2 \end{array}$$

43 tens – 31 tens = 12 tens

- 'Write "12" underneath the "31".'

Step 5 – exchange tens for ones and combine with the existing ones

$$\begin{array}{r} 0 \ 1 \\ 31 \overline{) 4 \ 3 \ 4} \\ \underline{3 \ 1} \quad \downarrow \quad (1 \text{ ten} \times 31 = 31 \text{ tens}) \\ 1 \ 2 \ 4 \end{array}$$

12 tens = 120 ones

120 ones + 4 ones = 124 ones

- 'Write "4" after the "12".'

Step 6 – divide the ones

$$\begin{array}{r} 0 \ 1 \ 4 \\ 31 \overline{) 4 \ 3 \ 4} \\ \underline{3 \ 1} \quad (1 \text{ ten} \times 31 = 31 \text{ tens}) \\ 1 \ 2 \ 4 \\ \underline{1 \ 2 \ 4} \quad (4 \text{ ones} \times 31 = 124 \text{ ones}) \end{array}$$

124 ones ÷ 31 = 4 ones

(refer to the ratio chart)

- 'Write "4" in the ones column of the answer line and write "124" underneath the "124", aligning the digits.'

Step 7 – subtract to show there is no remainder

$$\begin{array}{r} 0 \ 1 \ 4 \\ 31 \overline{) 4 \ 3 \ 4} \\ \underline{3 \ 1} \quad (1 \text{ ten} \times 31 = 31 \text{ tens}) \\ 1 \ 2 \ 4 \\ \underline{1 \ 2 \ 4} \quad (4 \text{ ones} \times 31 = 124 \text{ ones}) \\ 0 \end{array}$$

124 ones – 124 ones = 0 ones

- 'Write "0" underneath the "31".'

Year 6 – addition and subtraction

Application of Number Facts

Concrete and Pictorial Representations

Abstract

Number facts to 100,000,000
 (These are explored both as additive and multiplicative equations and applied within the range of strategies listed below)

1,000,000	
500,000	500,000

$$1,000,000 \div 2 = 500,000$$

$$1,000,000 \div 500,000 = 2$$

$$\frac{1}{2} \times 1,000,000 = 500,000$$

$$1,000,000 \times \frac{1}{2} = 500,000$$

$$2 \times 500,000 = 1,000,000$$

$$500,000 \times 2 = 1,000,000$$

1,000,000			
250,000	250,000	250,000	250,000

$$1,000,000 \div 4 = 250,000$$

$$1,000,000 \div 250,000 = 4$$

$$\frac{1}{4} \times 1,000,000 = 250,000$$

$$1,000,000 \times \frac{1}{4} = 250,000$$

$$4 \times 250,000 = 1,000,000$$

$$250,000 \times 4 = 1,000,000$$

1,000,000				
200,000	200,000	200,000	200,000	200,000

$$1,000,000 \div 5 = 200,000$$

$$1,000,000 \div 200,000 = 5$$

$$\frac{1}{5} \times 1,000,000 = 200,000$$

$$1,000,000 \times \frac{1}{5} = 200,000$$

Mental Strategies with numbers in the millions

Each of the mental strategies taught in Year 3 and used in Year 4 and 5 with increasingly larger numbers are applied and explored within numbers in the millions. This gives a chance for the children to be reminded of those strategies and to explore how patterns within the small place values can also be seen in larger numbers (for example, that $4 \times 250 = 1,000$ and $4 \times 250,000 = 1,000,000$).

See the Year 3 section above for an explanation and example of each mental strategy covered.

Column Addition and subtraction

The column addition and subtraction algorithms taught in Y3 and used in Year 4 and 5 with increasingly larger numbers are further extended into the millions.

For an explanation of these methods and how they are introduced, see the Y3 section above.

Column addition:

$$\begin{array}{r} 6\ 4\ 3,\ 8\ 0\ 1 \\ +\ 5\ 0\ 5,\ 3\ 7\ 0 \\ \hline 1,\ 1\ 4\ 9,\ 1\ 7\ 1 \\ \hline \end{array}$$

Year 6 – multiplication and division

By Year 6, children have already learnt all of the written methods for multiplication and division required for future learning and life: long and short multiplication and division. The emphasis in Year 6 is on developing a greater awareness of when each of the written and mental methods would be most efficient and reliable to use in a range of context. This includes applying each of their previously learnt strategies to decimals.